History of Science and Technology In Ancient India

— The Beginnings —

with a foreword by
JOSEPH NEEDHAM

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the geometrical principles usually followed by the "seal"-cutters.

3. The monumental remains of fortifications, granaries, public-baths, roads, house-blocks excavated at Harappa, Mohenjo-daro and recently also at Kalibangan and the remains of a dockyard at Lothal. Geometrical knowledge apart, we are obliged to deduce from these the knowledge of mathematics in other forms, like calculations of the number of workers, engineers, architects required for their construction and hence the amount of food etc. necessary for their subsistence.

4. Certain designs on Harappan potteries, like "squares inscribed in compus-drawn intersecting circles" appear to be pointers to the theory and practice of geometry.

Thus, notwithstanding the fact that no written document testifying to the mathematical activities of the Harappans is available for us, we have many evidences that remain inexplicable without the admission of mathematical activities on their part. It remains for us to see if any light is possibly thrown by what we propose to call "the method of retrospective probing", *i.e.* deductions from the mathematical documents of later times which are evidently in need of the assumption of antecedent historical development.

3. THE SCALES OF LENGTH MEASURE

We begin with a brief account of the scales or instruments for measuring length, fragmentary remains of which are so far recovered from the ruined sites of the Harappan Culture.

These, as we have already said, are three. Of these, one was first reported by Mackay in 1938 in his *Further Excavations at Mohenjo-daro*[2], the second first reported by Vats in 1953 in his *Excavations at Harappa*[3] and the third first reported in 1973 by Rao.[4]

2. Mackay FEM, Pl. cvi. 30; also pp 404-5
3. Vats EH Pl. CXXV, 39a and p. 365.
4 S. R. Rao, in IAR, 59-60, pl. xiii. b., also p. 17. cf also S. R. Rao, LIC 105: the one "found at Lothal is said to be more accurate, as the divisions marked on it (Fig. 28, pl xxxii A) are smaller than those marked on the scale from Mohenjo-daro. A graduated scale is reported from Kalibangan too, but details are not known".

According to the sites in which these are found, therefore, these three scales may be referred to as the Mohenjo-daro-scale, the Harappa-scale and the Lothal-scale. The physical descriptions of these scales, though oft-repeated in recent archaeological literature, may briefly be recapitulated.

The Mohenjo-daro-scale is made of shell, the Harappa-scale of bronze and the Lothal one of ivory. The selection of the materials has its own interest, not merely because of their durability but also of their comparative resistance to easy contraction or expansion due to temperature variation. Much more important for our own discussion, however, is the mathematical aspects of the scales—the unis of linear measures suggested by these.

The Mohenjo-daro scale first[5] :

It has nine graduations and there is a hollow circle on one graduation, while on the fifth graduation therefrom, there is a large circular dot. This sequence would indicate that a hollow circle would occur five graduations after the dot and so on. The graduation lines are made with a fine instrument and are uniformly thick and properly graded in length. The distance between two adjacent lines is, on an average, 6.7056 mm. The accuracy of the graduations is very high, as the mean error of graduation is 0.075 mm. Thus the length contained between five graduations is 33.528 mm and the length between one hollow circle and the next would be 67.056 mm. This length of 67.056 mm apparently constitutes the major graduation of the scale Since the sub-division of the major graduation of the scale is decimal, we may reasonably assume that the original scale had ten major graduations on it. This assumption leads us to the conclusion that the length of the total scale could be 670.56 mm. This is a very interesting result, as the total length of the scale is practically equal to two-thirds of a metre. It is also noteworthey that the length-scale appears to be decimally divided like the metre

We shall presently see that the evidence of the series of weights also found in the Harappan Culture corroborates the decimal system having been current there. But let us first note the system of length measures as evidenced by the scales. Mackay, to whom we owe the first report on the Mohenjo-daro scale, also drew our attention to the decimal system on which it was fashioned, and observed :[6]

5. Mainkar in FIC 146.
6. Mackay FEM, 404-5.

The decimal system of liner-measure is known in Egypt as early as the Fourth Dynasty, and a decimal division of the cubit in the Twelfth Dynasty has been noted by Kahun. Both the decimal and the sexagesimal systems were in use in early Sumer, though it is not yet known which came first According to Langdon, both systems were in use in Jemet Nasr; and on the Fara tablets, also, which must be dated to the Early Dynastic period, the two systems were used We are told, however, that a purely decimal system is found on the Proto-Elamite tablets, and it may be that it was from Elam that the system was introduced into N.W. India, though on the basis that every man has ten fingers it seems to me that the decimal system should be more primitive than the sexagesimal, and that it may have had independent origins

In view of the trade relations between the Harappans and Mesopotamians, diffusion of scientific ideas need not be necessarily ruled out, though in the absence of any positive evidence in favour of it, it seems safer not to indulge in conjectures. In any case, the systematic and uniform use of the decimal system in the Harappan Culture—about which we shall presently see more—is a prominent feature of the period of First Urbanization and it could have interesting consequences in the comparatively later period of Indian culture. For the present, however, let us pass on to the other scales found among the Indus ruins.

S. R. Rao, as we have already noted, has reported a truncated piece of ivory scale at Lothal :[7]

The Lothal scale is 15 mm broad, 6 mm thick and has a length of 128 mm. Only 27 graduation lines can be easily seen, the length over which these lines are spread being 46 mm. The average distance between the graduation lines is, therefore, 1.704 mm The sixth and the twentyfirst graduation lines are longer than the rest. The length between these two graduation lines is, by calculation, 25.56 mm

Is it, then, indicative of some different unit of length measure than was current in Mohenjo-daro? Mainkar[8] has answered this question and we may quote him at some length :

The Mohenjo-daro scale and the Lothal scale are apparently different but on analysis they prove to be practically equal. It will be noticed that 20 divisions of Lothal scale are equal to 34 mm, which is almost equal to the distance between the hollow circle and

7. Mainkar in FIC 146.
8. *Ibid.*

the circular dot on the Mohenjo-daro scale, namely 33.53 mm. This fact establishes that the two scales are related, but their division into smaller graduations are different. The smaller graduations of the Mohenjo-daro scale are at every 6.706 mm, while those of the Lothal scale are at 1.7 mm, the ratio being 4 Lothal graduations are equal to one graduation of the Mohenjo-daro scale. The smallness of the Lothal scale-graduation indicates that it was used for finer measurements.

One example of the need for such finer measurement is suggested by S. R. Rao: "The Indus seals can be measured more accurately in terms of the smaller divisions of the Lothal scale than in terms of the divisions of the Mohenjo-daro scale."[9] "This", observes Mainkar, "is a very plausible and satisfying explanation."[10]

Much more important for our immediate discussion are two other points sought to be established by Mainkar. These are: (1) His view of the possible use of the Indus Valley scales of linear measurement in brick-making and architectural techniques in the period of First Urbanization, and (2) His attempt to correlate the units of length measures with the same in later Indian history.

Before passing on to these—particularly the first of the two points—we may as well note the very legitimate caution with which he begins this discussion:[11]

> While comparing the theoretical and actual length of linear measures, it should be borne in mind that, even under modern conditions, it requires very costly machinery to draw lines to a thickness of 0.050 mm. Further, it is difficult to read, with the naked eye, graduation lines separated by a distance of less than half-a-milimetre It is, therefore, necessary to understand that when the calculated length of say, 68 mm, is equated with an actual distance of, say 67 mm, we may not be introducing a substantial error in our argument. Such equations have always to be treated as guidelines rather than strict mathematical formulations.

With this caution in mind, let us see how Mainkar wants to show that "the three scales discovered in three widely separated centres of the Indus Civilization are interrelated, and can

9. S. R. Rao, in LIC 107.
10. Mainkar in FIC 146.
11. *Ibid.*

explain statisfactorily the dimensions of the bricks, the baths, the dock and the like."[12]

The bricks first. Mainkar considers the following five "nominal sizes" of the bricks found in the different centres of the Indus Civilization :

1. Mohenjo-daro : 225×115×57 mm.
2. Lothal and Mohenjo-daro : 250×125×60 mm.
3. Lothal : 280×140×65 mm
4. Kalibangan and Mohenjo-daro : 300×150×75 mm.
5. Kalibangan : 400×200×100 mm.[13]

We have already noted that the standardization of the ratio of the three dimensions of the bricks—the length, breadth and thickness—as 4 : 2 : 1 is extremely significant from the viewpoint of efficient bonding, and it reminds us of the ratio of the sides of the standard bricks being used in our times. For the present let us note Mainkar's analysis of how the determination of the brick-sizes mentioned above appear to be indicative of the use of the Indus Valley scales. As he observes :[14]

It is obvious that the longest side of the Mohenjo-daro brick, namely 225 mm is nearly 9×25.6 mm. i e. 9 times the major graduation of the Lothal scale The longest side of the Lothal brick, namely 250 mm, is equal, within limits of error to 150 small graduations of the Lothal scale (1.7 mm. ot each) or 10 major graduations of 25.56 mm. each, while the longest side of the larger brick, namely 280 mm is made by the addition of one major graduation or 15 small graduations on the Lothal scale, i e. 11 large graduations (25 56×11 mm).

The brick with 300 mm side is a further extension of the above principle and comes to 180 Lothal small graduations or 12 large Lothal graduations (25.56×12 mm). In like manner the 400 mm brick would appear to be equal to 16 larger divisions of the Lothal scale (25.56×16 mm).

This analysis shows that perhaps, bricks were made in dimensions which were integral multiples of large graduations of the Lothal scale, namely 25.56 mm. The other dimensions of the bricks, being in the ratio of 1 : $^1/_2$: $^1/_4$ also fall into a rational number of Lothal graduations.

12. *Ibid* 147.
13. *Ibid.*
14. *Ibid.*

Admitting this, we have to accept that brick-making according to the application of some definite scale which we come across practically throughout the *Sulva-sutra*-s is indicative of a very ancient tradition, inasmuch as this tradition goes back to the period of the First Urbanization. This, in other words, means that the relation of mathematical calculation with brick-technology has a hoary past.

In view of the large number of brick-types mentioned in the Sulva texts each with very specific measurement in terms of the units of linear measures accepted by the texts, it would be a laborious process to try to assess the measurements of the Sulva bricks in terms of the scales of the Harappan Culture. Besides, that is not necessary for our main argument, namely that it is not *prima facie* impossible to try to trace the tradition of the application of mathematical calculation to brick technology to the ancient Harappan Culture. This tradition, once accepted, may explain the meticulous care taken by the brick-makers of our Sulva texts to be specific or accurate about the measures of the brick-types, a large number of which they had to improvise in order to meet the requirements of the peculiar structures they were asked to execute. Incidentally, this technique of improvising new and newer brick-types, too, could have its roots in the Harappan culture, where, apart from the standardised bricks, we also meet with various other brick-types, like the T-shaped one assumed as needed for covering the drains and the wedge-shaped bricks used for the construction of wells, drains or the grinding floor of the granaries.

But there is another point of considerable interest which may as well be noted in this connection. In spite of various conjectures, the fact remains that we have no definite knowledge of the language of the peoples in the Indus Valley Civilization. It is, therefore, futile to speculate on the possible terminologies used by the Harappan peoples for the units of length measures. In the history of Indian culture, the earliest evidences for such terminologies are to be found in the *Sulva-sutra*-s and *Arthasastra*. In both, the basic unit for length measure is called an *angula*, literally 'the finger'. For the sake of precision, however, the *Arthasastra* defines it as "the maximum width of the middle (part) of the middle finger of a middling man."[15] Whether the unit *angula* of the *Arthasastra* is exactly the same as understood in the *Sulva-sutra*-s may be open to some discussion, for the *Arthasastra*[16] proposes to measure it in terms of eight *yavamadhya*-s (the width of the middle of eight *yava*-s) whereas the *Baudhayana Sulva-sutra*[17] conceives it in terms of fourteen grains of the *anu* plant (understood by Thibaut as *panicum miliaceum*). But we may note here one point of some interest. According to both the texts[18] the longer unit called *aratni* (losely translated as 'cubit' by Kangle) is conceived in terms of 24 *angula*-s and it is also the same according to Yallaya's explanation of Aryabhata[19] though the latter uses the word *hasta* instead of *aratni* (literally the length from the elbow to the tip of the little finger). In any case, the fact is that the term *angula* stands for the basic unit of length measure in later literature, inclusive of the Sulva texts and there are at least some hints suggesting correlation between the *angula* of the *Baudhayana Sulva-sutra* and of the *Arthasastra* as well as of much later astronomical works.

Earlier writers like J. F. Fleet[20] were satisfied by roughly equating the *angula* to 3|4th of an inch, which makes it 19.499 mm. On the basis of a more meticulous calculation, however, Mainkar equates the length of the *Arthasastra angula* to 17.78 mm. This gives a very interesting clue to correlate the basic unit of later linear measure, viz. the *angula*, to the length measure of the Lothal scale. As Mainkar[21] puts it :

> The author has shown, (in his articles) tracing the development of length and area-measures in India, that the *angula* which is the basic unit of length measures, mentioned in the *Arthasastra*, is 17.78 mm. This value is so nearly equal to the value of ten small graduations of the Lothal scale (1.703×10 mm), that they may be considered as being practically equal. If this is accepted, and Rao agrees with it, the entire series of length-measures specified in the *Arthasastra* falls in a pattern with the Indus scales. The author has shown in his articles mentioned above, that the length-measures used in India throughout later periods were related in some manner or other, with the length-measures specified in the *Arthasastra*. It is,

15. *Arthasastra*, ii. 20.7.
16. *Ibid* ii. 20.5.
17. *Baudh Sul Su*. i. 4.
18. *Arthasastra*, ii. 20. 12 ; and *Baudh. Sul Su*. i. 16.
19. Shukla and Sarma, *Aryabhatiya*, intro xliii.
20. J. F. Fleet in JRAS, 1912. 233.

therefore, possible to assert that the Indus length-measures had a very profound influence on the length-measures used in India up to a few years back.

S. R. Rao wants to go a step further :[22]

It appears that both 'foot' and 'cubit' were treated as units for linear measures. The 'foot' is said to be of 13.2 ins. (33.5 cms.) and the 'cubit' varying between 20.3 and 20.8 ins. (51.5 and 52.8 cms). The houses in Lothal can be measured in terms of complete units of 'foot', e.g. House No. 159 (phase IV A) measures 40×20 units, and warehouse 117×123 units, the unit in each case being 13.2 ins.

But before passing on to see more of the application of mathematics to the brick-structures of the Harappan Culture, we may ask ourselves a simple question. Could it be that the correlation of the *angula* of the later texts inclusive of the Sulva-s with the linear measure of the Indus scales be itself an indication that wants us to seek the roots of the Sulva mathematics in the mathematical activities in the First Urbanization?

4. BRICK-TECHNOLOGY AND MATHEMATICS IN FIRST URBANIZATION

While analysing the *Sulva-sutra*-s we were led to the view that the mathematics codified in these texts is inconceivable without the tradition of highly sophisticated brick-technology. The texts give us the impression that this mathematical knowledge was above all the outcome to meet the theoretical requirements of the brick-makers, brick-layers, architects and other technicians, who were required to execute the construction of certain specified forms of brick-structures. At the same time, we were confronted with an apparently anomalous situation. The texts cannot but be placed in a period which, archaeologically speaking, was unaware of any sophisticated brick-technology. Hence we were led to raise the question concerning the possible roots of this mathematics in the mathematical activities of the First Urbanization, one of the most conspicuous features of which had been highly sophisticated brick-technology. But the first point that requires to be established before answering the

21. Mainkar in FIC 147-48.
22. S. R. Rao, LIC 107.

question is that we have definite evidences indicating mathematical activities in Harappan Culture. We have just seen that the linear measures of the broken scales found among the ruins of the First Urbanization appear to foreshadow the basic unit of the linear measure assumed by the Sulva texts. We now pass on to discuss how far we are obliged to presume the application of mathematical knowledge from the remains of the brick-structures of the First Urbanization.

In default of anything directly documenting mathematical activities during the period of the First Urbanization, it is impossible of course for us to hope to have any systematic knowledge of Indus mathematics.

Nevertheless, it is important to note that serious archaeologists and other scholars have felt obliged to argue that the brick-structures of the First Urbanization remain unexplained without the assumption of the application of a good deal of mathematical knowledge.

Here is how M. N. Deshpande[23] puts the general argument .

> The monumental remains of fortifications, granaries, public-baths. roads and house-blocks excavated at Harappa. Mohenjo-Daro and recently at Kalibangan and remains of dockyard at Lothal imply a good deal of arithmetic and knowledge of geometry. Keeping of accounts for the construction of public buildings such as of labour and material would entail complicated calculations Unfortunately, direct evidence of such accounting is not available. As regards the knowledge of geometry besides the few measuring rods and other instruments which have come to light we have largely to depend for such deductions on the data supplied by the buildings themselves It is obvious from the meticulous care the Harappans took in planning the city with well laid-out streets that they knew fundamentals of surveying. This would include knowledge of levelling as without detailed measurements it would not have been possible to plan the sewage system. The use of standardized bricks having plain rectangular faces, parallel sides, sharp, straight, right-angled edges including wedge-shaped bricks in the construction of circular wells so as to produce the inner and outer circumference would presuppose knowledge of geometry of parallels and circles.

Mainkar with his illuminating analysis of the correlation of the standard brick-sizes of the Indus ruins with linear mea-

23. M. N. Deshpande, in IJHS VI. No. I: 9.

sures of the Indus scales, gives us some specific examples of the mathematical calculations implied by a number of the brick-structures of the First Urbanization. We quote him at some length :[24]

Turning to other sources of measurement the 'Great Bath' at Mohenjo-daro had average measurements of 11.89 m. × 7.01 m. with a depth of 2.44 m. Adjoining the 'Great Bath' were smaller rooms of 2.9 m. × 1.8 m. each. It is apparent that each of the smaller rooms had approximately ¼ of the average measurements of the Great Bath' (2 9×4=11.6 and 1.8×4=7.2). In their turn, the measurements of the smaller bathrooms are related to the bricks having 300 mm as the longest dimension. Thus, the longer side of the bath-rooms is 10 times and the smaller side 6 times the longest side of the 300 mm brick. The longer side of the 'Great Bath' is equal to a length of 40 bricks and the smaller side to a length of 24 bricks the depth being equal to the length of 8 bricks.

Rao has recorded that the average measurements of the dock at Lothal are 214 m × 36 m, the foundations being 1.78 m wide and the walls above the ground 1.04 m. These measurements are interesting because 1.04 m is equal to 40 large graduations of Lothal scale (25.56 mm), while 1.78 m is equal to 1000 times the small graduations of (1.7 mm) of the Lothal scale or $66^2/_3$ large graduations of 25.56 mm. The major dimensions of the dock are in the proportion of 6 : 1. The dimension of 36 metres is 20 times the width of the foundations, namely 1.78 m. The latter value is in turn related to the small graduation of the Lothal scale by 1,000 times

The width of the doors in the houses at Mahenjo-daro was about 1.02 m.; this is 40 times 25.56 mm. the large division of the Lothal scale or 15 times the large division of Mohenjo-daro scale, namely 67 06 mm.

There are numerous other dimensions and measurements which could be analysed. But, it is considered that the examination carried out so far is adequate for indicating that the three scales discovered in three widely separated centres of the Indus Civilization are inter-related, and can explain satisfactorily the dimensions of the bricks, the baths, the dock and the like.

All this may be taken as pointers to a very interesting line of research requiring the co-operation of archaeologists, architects and mathematicians for determining the nature of mathematical knowledge necessarily presupposed by the structural remains as well as by the decorative designs of the Indus relics. R. P. Kulkarni of the Maharashtra Enginnering Research

24. Mainkar in FIC, 147.