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A NOTE ON ECONOMETRICS OF JOINT PRODUCTION

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IN A RECENT PAPER in *Econometrica*, (April, 1968), Vinod [**3**] suggested a canonical correlations technique for estimating joint production processes. Estimates from such a procedure have economic interpretation only under certain restrictive assumptions, and the purpose of this note is to point out those restrictions.

Joint production includes two cases: (i) when there are *multiple products*, each produced under *separate* production process, for example, wheat and livestock; and (ii) when there is *joint production*, i.e., when several outputs are produced from a single production process, for example, sugar and molasses.

The case of multiple products is a problem of aggregation. Let the production functions producing outputs y_1 and y_2 with the inputs x_1 and x_2 be

$$\begin{aligned} \log y_1 &= a_{11} \log x_{11} + a_{12} \log x_{12}, \\ \log y_2 &= a_{21} \log x_{21} + a_{22} \log x_{22}, \end{aligned} \tag{1}$$

where x_{ij} is the amount of the jth input used in producing the ith output, and each of these equations specifies a production process.

Fitting the equation

$$c_1 \log y_1 + c_2 \log y_2 = d_1 \log x_1 + d_2 \log x_2 \tag{2}$$

via canonical correlations implies the following specification

$$\begin{aligned} \log y_1 &= b_{11} \log x_1 + b_{12} \log x_2, \\ \log y_2 &= b_{21} \log x_1 + b_{22} \log x_2, \end{aligned} \tag{3}$$

where x_1 and x_2 are the aggregate inputs.

There is correspondence between parameters in the specifications (1) and (3) only under certain restrictive economic assumptions. One specification allowing for correspondence of the parameters is when each input used in each output is a constant proportion of its corresponding aggregate. In this case parameters in (3) are linear functions of parameters in (1). Griliches [**1**] has discussed some problems associated with estimating this specification.

The case of joint production is a technological phenomenon, not merely a problem in aggregation. In joint production all products are produced in one production process which may be specified in implicit form as $F(y_1, y_2, x_1, x_2) = 0$. For estimation purposes we may restrict the function to satisfy the functional form $f(y_1, y_2) = g(x_1, x_2)$. These functions may be interpreted as follows: A firm produces some abstract good—juice of sugar cane, in our example—using inputs to minimize total cost, and transforms the abstract good into outputs to maximize total revenue. We may interpret $f(y_1, y_2)$ as a transformation curve and $g(x_1, x_2)$ as a joint production function.

For these functions to be economically meaningful, the production function should be convex and the transformation curve concave.

Fitting the equation (2) in canonical correlations implies that the transformation curve is of the form

$$y_1^{c_1} y_2^{c_2} = g.$$

This function is nondecreasing and concave only when the parameters are nonnegative and the matrix of second derivatives is positive. It can be shown very easily by differentiating the function that for no values of the parameters is the above function nondecreasing and concave. Therefore, Mundlak [**2**] introduced a transcendental component in the transformation curve.

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One function which would satisfy the economic restrictions, as a limiting case, is the linear model, for which both the production function and the transformation curve are perfectly elastic.

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