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On Estimation of Demand and Supply Equations of a Commodity

ABSTRACT

This paper develops Modified Two Stage Least Squares (MTSLS) estimation procedure to estimate demand and supply equations of a commodity. The MTSLS estimates are asymptotically unbiased and have variance smaller than or equal to those of Two Stage Least Squares (TSLS) for any sample size. The gain in precision is derived from exploiting information on the identifying variables. The MTSLS shares computational simplicity with the TSLS.

1. Introduction

In a simple equilibrium market clearing model the price and quantity sold are jointly determined by the forces of demand and supply. In order to empirically estimate the demand and supply equations of a particular commodity we normally specify the underlying economic mechanism by a simultaneous equations model and estimate it by Two Stage Least Squares (TSLS)[1]. In this paper we suggest a modification to the TSLS procedure which improves the precision of the parameter estimates of the model. The modification is also applicable to other simultaneous equations models with only two structural equations having the same dependent variable.

Let the market clearing mechanism for a commodity be specified by the following simultaneous equation model:

$$\left.\begin{aligned} y_t^d &= \alpha p_t + a_1 w_{1t} + \ldots + a_k w_{kt} + c_1 x_{1t} + \ldots + c_n x_{nt} + \epsilon_t^d \\ y_t^s &= \beta p_t + b_1 w_{1t} + \ldots + b_k w_{kt} + d_1 z_{1t} + \ldots + d_m z_{mt} + \epsilon_t^s \\ y_t^d &= y_t^s \qquad t = 1, \ldots, T \end{aligned}\right\} \quad (1)$$

where y is the commodity exchanged, p is the price of the commodity, and w's, x's and z's are exogenous variables. The exogenous variables

1. The TSLS procedure is due to Theil (3). The TSLS estimates are consistent estimates.

w's are common to both the demand and supply equations. In this model all the variables are expressed as deviations from their respective means, hence the constant terms are implicit. We assume that the error terms (ϵ's) are homoscedastic and serially independent. When there is no ambiguity we shall drop the subscript t.

The TSLS estimates of the parameters of model (1) are obtained as follows.

In the first stage the reduced form equation for p is obtained as

$$p = \pi_1 x_1 + \ldots + \pi_n x_n + \pi_{n+1} z_1 + \ldots + \pi_{n+m} z_m + \lambda_1 w_1 + \\ + \ldots + \lambda_k w_k + \epsilon_2, \qquad (2)$$

and estimated by Ordinary Least Squares (OLS)[2]. In the second stage the demand and supply equations are estimated by computing the following regression equations again using OLS:

$$y = \hat{\alpha}\hat{p} + \hat{a}_1 w_1 + \ldots + \hat{a}_k w_k + \hat{c}_1 x_1 + \ldots + \hat{c}_n x_n + e_3 \qquad (3)$$

$$y = \hat{\beta}\hat{p} + \hat{b}_1 w_1 + \ldots + \hat{b}_k w_k + \hat{d}_1 z_1 + \ldots + \hat{d}_m z_m + e_4 \qquad (4)$$

where

$$\hat{p} = \hat{\pi}_1 x_1 + \ldots + \hat{\pi}_n x_n + \hat{\pi}_{n+1} z_1 + \ldots + \hat{\pi}_{n+m} z_m + \\ \hat{\lambda}_1 w_1 + \ldots + \hat{\lambda}_k w_k$$

with the $\hat{\lambda}$'s and $\hat{\pi}$'s being the OLS estimates from the reduced form equation (2).

The TSLS estimation procedure computes α and β separately from equations (3) and (4) rather than simultaneously, which leads to the often raised criticism that the TSLS estimates are *limited information* estimates. In the present model it is possible to estimate α and β jointly.

2. The Modified Two Stage Least Squares

Consider the reduced form equation for y from the model (1) which may be written as

$$y = \beta(\pi_1 x_1 + \ldots + \pi_n x_n) + \alpha(\pi_{n+1} z_1 + \ldots + \pi_{n+m} z_m) \\ + \mu_1 w_1 + \ldots + \mu_k w_k + \epsilon_5, \qquad (5)$$

2. This reduced form equation and hence the TSLS estimates, are possible only when $T > n + m + k$.

the reduced form equation for y separates the exogenous variables into three groups; the variables identifying the demand equation, the z's; the variables identifying the supply equation, the x's; and the common variables, the w's.

The reduced form equation for y incorporates information on all the equations of the model by separating the variables into the three groups. Estimating α and β from equation (5) would provide joint estimates of these parameters using all the available information on the model. In order to estimate equation (5) we need the values of π's which are unknown. However, analogous to the TSLS estimation procedure we may use the OLS estimates of π's from equation (2) in estimating α and β from equation (5). Estimation of the following equation by OLS would thus provide our Modified Two Stage Least Squares (MTSLS) estimates of α and β:

$$y = \beta^* \hat{x} + \alpha^* \hat{z} + \mu_1^* w_1 + \ldots + \mu_k^* w_k + e_6, \tag{6}$$

where $\hat{x} = \hat{\pi}_1 x_1 + \ldots + \hat{\pi}_n x_n,$

and $\hat{z} = \hat{\pi}_{n+1} z_1 + \ldots + \hat{\pi}_{n+m} z_m.$

Having obtained the MTSLS estimates α^* and β^* from equation (6) we may estimate the rest of the parameters of the model by OLS estimation of the following equations:

$$(y - \alpha^* \hat{p}) = a_1^* w_1 + \ldots + a_k^* w_k + c_1^* x_1 + \ldots + c_n^* x_n + e_7 \tag{7}$$

$$(y - \beta^* \hat{p}) = b_1^* w_1 + \ldots + b_k^* w_k + d_1^* z_1 + \ldots + d_m^* z_m + e_8 \tag{8}$$

where the estimates with asterick (*) are the MTSLS estimates of the corresponding parameters.

3. Properties of MTSLS Estimates

The MTSLS estimates are asymptotically unbiased and have smaller or equal variances compared to the corresponding TSLS estimates for any sample size. The proofs are provided in the rest of this paper.

In order to establish the distributional properties of the MTSLS estimates let us compare the following two equations estimated by OLS:

$$y = \tilde{\alpha}\hat{z} + \tilde{a}_1 w_1 + \ldots + \tilde{a}_k w_k + \tilde{c}_1 x_1 + \ldots + \tilde{c}_n x_n + e_9 \quad (9)$$

$$y = \hat{\alpha}(\hat{z} + \hat{\lambda}_1 w_1 + \ldots + \hat{\lambda}_k w_k + \hat{\pi}_1 x_1 + \ldots + \hat{\pi}_n x_n) + \hat{a}_1 w_1 + \ldots + \hat{a}_k w_k + \hat{c}_1 x_1 + \ldots + \hat{c}_n x_n + e_3, \quad (3a)$$

where equation (3*a*) is a restatement of equation (3) yielding the TSLS estimate of α. In a linear regression equation addition of a constant multiple of an independent variable to another independent variable does not alter the implicit OLS estimates of a parameter.[3]

Therefore the OLS estimates of equations (9) and (3*a*) satisfy the following arithmetic identities:

$$\tilde{\alpha} \equiv \hat{\alpha}$$

$$\tilde{a}_i \equiv \hat{a}_i + \hat{\alpha}\hat{\lambda}_i, \qquad i = 1, \ldots, k$$

$$\tilde{c}_j \equiv \hat{c}_j + \hat{\alpha}\hat{\lambda}_j, \qquad j = 1, \ldots, n.$$

Whether we estimate equation (9) or (3*a*) we would obtain the same value for the estimate of α. Hence the statistical properties of $\hat{\alpha}$ and $\tilde{\alpha}$ are identical.

Now let us compare the OLS estimates of the following two equations:

$$y = \hat{\alpha}\hat{z} + \tilde{a}_1 w_1 + \ldots + \tilde{a}_k w_k + \tilde{c}_1 x_1 + \ldots + \tilde{c}_n x_n + e_9 \quad (9a)$$

$$y = \alpha^* \hat{z} + \beta^*(\hat{\pi}_1 x_1 + \ldots + \hat{\pi}_n x_n) + \mu_1^* w_1 + \ldots + \mu_k^* w_k + e_6 \quad (6a)$$

where equations (9*a*) and (6*a*) are restatements of equations (9) and (6) respectively. For analytical convenience we may interpret equation (6*a*) as a *restricted least squares* estimate of equation (9*a*) with the following restrictions imposed on it

$$c_i = (\hat{\pi}_i / \hat{\pi}_1)\, c_1, \qquad i = 2, \ldots, n.$$

3. This theorem is originally due to Frisch and Waugh (1). See also Tintner (4), pp. 301-7.

The distributional properties of restricted least squares estimates are studied in Goldberger (2). When a set of linear restrictions are imposed on a linear regression equation, the restricted estimates have the same extent of bias as that of the unrestricted estimates provided the restrictions are true, and the variance of a restricted estimate is smaller than the variance of the corresponding unrestricted estimate irrespective of the truth of the restrictions.[4]

Asymptotically the restrictions imposed on equation (6*a*) are true, because the $\hat{\pi}$'s are consistent estimates of the π's and equation (6*a*) is merely a reduced form equation. Hence both the estimates (α^* and $\hat{\alpha}$) have the same asymptotic bias. Since the TSLS estimates are asymptotically unbiased, the MTSLS estimate α^* is also asymptotically unbiased. Since the variance of the restricted estimate cannot exceed the variance of the unrestricted estimate irrespective of the truth of the restrictions it follows that for all sample sizes,

$$V(\alpha^*) \leqslant V(\hat{\alpha}).$$

By similar reasoning we may prove that the MTSLS estimate β^* is asymptotically unbiased and has smaller or equal variance compared to the corresponding TSLS estimate $\hat{\beta}$.

Now let us turn to the distributional properties of the MTSLS estimates of the coefficients of the exogenous variables in model (1). Consider the OLS estimates of the following equations:

$$(y_t - \hat{\alpha}\hat{p}_t) = \tilde{\tilde{\alpha}}\hat{p}_t + \tilde{\tilde{a}}_1 w_{1t} + \ldots + \tilde{\tilde{a}}_k w_{kt} + \tilde{\tilde{c}}_1 x_{1t} + \ldots + \tilde{\tilde{c}}_n w_{nt} + e_{10} \tag{10}$$

which may also be written in the matrix notation as

$$(Y - \hat{\alpha}\hat{P}) = \hat{P}\tilde{\tilde{\alpha}} + W\tilde{\tilde{a}} + X\tilde{\tilde{c}} + E \tag{10a}$$

where the capital letters correspond to vectors or matrices of observations of the corresponding small letters, and *a* and *c* are vectors of parameters.

4. See Goldberger [2], pp. 256-8.

The OLS estimates of equation (10*a*) are computed as

$$\begin{bmatrix} \tilde{\tilde{\alpha}} \\ \tilde{\tilde{a}} \\ \tilde{\tilde{c}} \end{bmatrix} = \begin{bmatrix} \hat{P}'\hat{P} & \hat{P}'W & \hat{P}'X \\ W'\hat{P} & W'W & W'X \\ X'\hat{P} & X'W & X'X \end{bmatrix}^{-1} \begin{bmatrix} \hat{P}'(Y-\hat{\alpha}\hat{P}) \\ W'(Y-\hat{\alpha}\hat{P}) \\ X'(Y-\hat{\alpha}\hat{P}) \end{bmatrix}$$

$$= \begin{bmatrix} \hat{P}'\hat{P} & \hat{P}'W & \hat{P}'X \\ W'\hat{P} & W'W & W'X \\ X'\hat{P} & X'W & X'X \end{bmatrix} \begin{bmatrix} \hat{P}'Y \\ W'Y \\ X'Y \end{bmatrix}$$

$$- \hat{\alpha} \begin{bmatrix} \hat{P}'\hat{P} & \hat{P}'W & \hat{P}'X \\ W'\hat{P} & W'W & W'X \\ X'\hat{P} & X'W & X'X \end{bmatrix}^{-1} \begin{bmatrix} \hat{P}'\hat{P} \\ \hat{P}'W \\ \hat{P}'X \end{bmatrix}$$

$$= \begin{bmatrix} \hat{\alpha} \\ \hat{a} \\ \hat{c} \end{bmatrix} - \hat{\alpha} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \hat{a} \\ \hat{c} \end{bmatrix}.$$

This result indicates that estimation of the following equation by the OLS procedure would yield exactly the same estimates as that of the TSLS for the parameters a's and c's:

$$(y-\hat{\alpha}\hat{p}) = \hat{a}_1 w_1 + \ldots + \hat{a}_k w_k + \hat{c}_1 x_1 + \ldots + \hat{c}_n x_n + e_{11}. \qquad (11)$$

Equation (11) yields the TSLS estimates of a's and c's and equation (7) yields the corresponding MTSLS estimates. In order to compare the statistical properties of these two estimates consider the following regression equation:

$$y = \alpha\hat{p} + a_1 w_1 + \ldots + a_k w_k + c_1 x_1 + \ldots + c_n x_n + e_{12}. \qquad (12)$$

For analytical convenience we may interpret equation (11) as a restricted least squares estimate of equation (12) where α is restricted to be $\hat{\alpha}$. Similarly equation (7) may be interpreted as a restricted least squares estimation of equation (12) where α is restricted to α^*.

From the properties of restricted least squares we know that restricted least squares estimates based on an unbiased estimate are also unbiased.[5] Since both the estimates α^* and $\hat{\alpha}$ are asymptotically unbiased estimates of α, the estimates (a^* and c^*) and ($\hat{a}$ and $\hat{c}$) are also asymptotically unbiased estimates of the corresponding parameters.

Since α^* has smaller variance than or equal to the variance of $\hat{\alpha}$, the restricted estimates based on α^* would have variance smaller than or equal to the variance of the restricted estimates based on $\hat{\alpha}$.[6] That is

$$V(a_i^*) \leqslant V(\hat{a}_i) \qquad i = 1, \ldots, k,$$

and

$$V(c_j^*) \leqslant V(\hat{c}_j) \qquad j = 1, \ldots, n.$$

By similar reasoning we may prove that the MTSLS estimates of the b's and d's are asymptotically unbiased and have variance smaller than or equal to the variance of the corresponding TSLS estimates.

Thus we have established that the MTSLS estimates of model (1) have the same desirable properties as TSLS and in addition have variances smaller than or equal to those of the corresponding TSLS estimates for any sample size. It may be seen from our analysis that the gain in MTSLS estimates is derived from the overidentifying variables of a parameter. When all parameters of the model are exactly identified, then, of course, there is no gain and both the TSLS and MTSLS yield the same estimates.

4. Conclusion

In empirical research we often estimate simultaneous equations systems involving a demand and a supply equation of a commodity. When some or all parameters of this system are overidentified it is possible to estimate them by the Modified Two Stage Least Squares. The MTSLS estimates are asymptotically unbiased and have variances smaller than or equal to the variances of the corresponding TSLS estimates for any

5. See Goldberger [2], pp. 258-9.
6. See Goldberger [2], p. 259.

sample size. The MTSLS estimation procedure shares computational simplicity with the TSLS.

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